**Problem 1:**

**Given:**  
f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7

**Question:**  
Is f(n)f(n)f(n) **O(n²)**?

**Solution:**

We want to determine if f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7 is **O(n²)**.

By the definition of Big-O notation, we say that a function f(n)f(n)f(n) is **O(g(n))** if there exist constants ccc and n0n\_0n0​ such that:

∣f(n)∣≤c⋅g(n) for all n≥n0|f(n)| \leq c \cdot g(n) \text{ for all } n \geq n\_0∣f(n)∣≤c⋅g(n) for all n≥n0​

In this case, we want to check if f(n)f(n)f(n) is **O(n²)**. Let’s break down f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7.

* The term 3n23n^23n2 dominates as nnn grows large because it grows faster than the linear term 5n5n5n and the constant term 777.
* Hence, for large nnn, f(n)f(n)f(n) behaves like 3n23n^23n2, which is O(n2)O(n^2)O(n2).

Thus, we can express:

f(n)=3n2+5n+7isO(n2)f(n) = 3n^2 + 5n + 7 \quad \text{is} \quad O(n^2)f(n)=3n2+5n+7isO(n2)

This is correct because 3n23n^23n2 dominates for large nnn, and the other terms do not affect the asymptotic complexity.

**Answer:**  
Yes, f(n)f(n)f(n) is **O(n²)**.

**Problem 2:**

**Given:**  
f(n)=2n+n3f(n) = 2^n + n^3f(n)=2n+n3

**Question:**  
Find g(n)g(n)g(n) such that f(n)f(n)f(n) is **O(g(n))**.

**Solution:**

We want to find a function g(n)g(n)g(n) such that f(n)=2n+n3f(n) = 2^n + n^3f(n)=2n+n3 is **O(g(n))**.

* The function f(n)=2n+n3f(n) = 2^n + n^3f(n)=2n+n3 consists of two terms: 2n2^n2n (exponential) and n3n^3n3 (polynomial).
* As nnn grows large, the exponential term 2n2^n2n will grow much faster than the polynomial term n3n^3n3. This means that for large nnn, the term 2n2^n2n will dominate the behavior of f(n)f(n)f(n).

Thus, f(n)f(n)f(n) is asymptotically dominated by 2n2^n2n, and we can say:

f(n)=O(2n)f(n) = O(2^n)f(n)=O(2n)

This means that g(n)=2ng(n) = 2^ng(n)=2n.

**Answer:**  
f(n)f(n)f(n) is **O(2ⁿ)**.

**Problem 3:**

**Given:**  
f(n)=5log⁡n+2nf(n) = 5 \log n + 2nf(n)=5logn+2n

**Question:**  
Is f(n)f(n)f(n) **O(n)**?

**Solution:**

We are tasked with determining whether f(n)=5log⁡n+2nf(n) = 5 \log n + 2nf(n)=5logn+2n is **O(n)**.

* The function f(n)=5log⁡n+2nf(n) = 5 \log n + 2nf(n)=5logn+2n consists of two terms: 5log⁡n5 \log n5logn (logarithmic) and 2n2n2n (linear).
* As nnn grows large, the linear term 2n2n2n will eventually dominate the logarithmic term 5log⁡n5 \log n5logn because linear growth outpaces logarithmic growth.

Therefore, for large nnn, the function behaves like 2n2n2n, which is **O(n)**.

We can say that f(n)f(n)f(n) is indeed **O(n)** because the linear term 2n2n2n dominates the logarithmic term as nnn increases.

**Answer:**  
Yes, f(n)f(n)f(n) is **O(n)**.

**Summary of Answers:**

1. f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7 is **O(n²)**.
2. f(n)=2n+n3f(n) = 2^n + n^3f(n)=2n+n3 is **O(2ⁿ)**.
3. f(n)=5log⁡n+2nf(n) = 5 \log n + 2nf(n)=5logn+2n is **O(n)**.

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**Big Omega Practice Problems**

**Problem 1:**

**Given:**  
f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7

**Question:**  
Is f(n)f(n)f(n) **Ω(n²)**?

**Solution:**

We want to determine if f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7 is **Ω(n²)**.

By the definition of **Big Omega**, we say that f(n)f(n)f(n) is **Ω(g(n))** if there exist constants ccc and n0n\_0n0​ such that:

f(n)≥c⋅g(n)for alln≥n0f(n) \geq c \cdot g(n) \quad \text{for all} \quad n \geq n\_0f(n)≥c⋅g(n)for alln≥n0​

In this case, we want to check if f(n)=3n2+5n+7f(n) = 3n^2 + 5n + 7f(n)=3n2+5n+7 is **Ω(n²)**.

* The term 3n23n^23n2 dominates as nnn grows large.
* We can bound f(n)f(n)f(n) below by 3n23n^23n2 for sufficiently large nnn.

For large nnn, the function behaves like 3n23n^23n2, and thus f(n)f(n)f(n) is lower-bounded by n2n^2n2. This means that f(n)f(n)f(n) is **Ω(n²)**.

**Answer:**  
Yes, f(n)f(n)f(n) is **Ω(n²)**.

**Problem 2:**

**Given:**  
f(n)=n3−2n2+10f(n) = n^3 - 2n^2 + 10f(n)=n3−2n2+10

**Question:**  
Is f(n)f(n)f(n) **Ω(n³)**?

**Solution:**

We want to determine if f(n)=n3−2n2+10f(n) = n^3 - 2n^2 + 10f(n)=n3−2n2+10 is **Ω(n³)**.

* The dominant term in f(n)f(n)f(n) is n3n^3n3, as n3n^3n3 grows faster than both n2n^2n2 and the constant 101010 as nnn becomes large.
* We can observe that for large nnn, the n3n^3n3 term dominates the behavior of f(n)f(n)f(n).

Thus, f(n)f(n)f(n) is **Ω(n³)** because the function grows at least as fast as n3n^3n3 for large nnn.

**Answer:**  
Yes, f(n)f(n)f(n) is **Ω(n³)**.

**Problem 3:**

**Given:**  
f(n)=5n+2n+1f(n) = 5n + 2\sqrt{n} + 1f(n)=5n+2n​+1

**Question:**  
Find g(n)g(n)g(n) such that f(n)f(n)f(n) is **Ω(g(n))**.

**Solution:**

We want to find g(n)g(n)g(n) such that f(n)=5n+2n+1f(n) = 5n + 2\sqrt{n} + 1f(n)=5n+2n​+1 is **Ω(g(n))**.

* The function consists of three terms: 5n5n5n, 2n2\sqrt{n}2n​, and 111.
* As nnn grows large, the linear term 5n5n5n will dominate the other terms, because nnn grows faster than n\sqrt{n}n​ or a constant.

Thus, we can say that f(n)f(n)f(n) is **Ω(n)** because the linear term 5n5n5n dominates and grows faster than the other terms.

**Answer:**  
f(n)f(n)f(n) is **Ω(n)**.

**Big Theta Practice Problems**

**Problem 1:**

**Given:**  
f(n)=4n2+3n+5f(n) = 4n^2 + 3n + 5f(n)=4n2+3n+5

**Question:**  
Is f(n)f(n)f(n) **Θ(n²)**?

**Solution:**

We want to determine if f(n)=4n2+3n+5f(n) = 4n^2 + 3n + 5f(n)=4n2+3n+5 is **Θ(n²)**.

By the definition of **Big Theta**, we say that f(n)f(n)f(n) is **Θ(g(n))** if there exist constants c1c\_1c1​, c2c\_2c2​, and n0n\_0n0​ such that:

c1⋅g(n)≤f(n)≤c2⋅g(n)for alln≥n0c\_1 \cdot g(n) \leq f(n) \leq c\_2 \cdot g(n) \quad \text{for all} \quad n \geq n\_0c1​⋅g(n)≤f(n)≤c2​⋅g(n)for alln≥n0​

In this case, we want to check if f(n)=4n2+3n+5f(n) = 4n^2 + 3n + 5f(n)=4n2+3n+5 is **Θ(n²)**.

* The dominant term in f(n)f(n)f(n) is 4n24n^24n2, and for large nnn, the 3n3n3n and 555 terms become negligible.
* Therefore, f(n)f(n)f(n) grows asymptotically like 4n24n^24n2, which is **Θ(n²)**.

**Answer:**  
Yes, f(n)f(n)f(n) is **Θ(n²)**.

**Problem 2:**

**Given:**  
f(n)=6nlog⁡n+10nf(n) = 6n \log n + 10nf(n)=6nlogn+10n

**Question:**  
Is f(n)f(n)f(n) **Θ(n \log n)**?

**Solution:**

We want to determine if f(n)=6nlog⁡n+10nf(n) = 6n \log n + 10nf(n)=6nlogn+10n is **Θ(n \log n)**.

* The first term 6nlog⁡n6n \log n6nlogn is the dominant term because nlog⁡nn \log nnlogn grows faster than nnn as nnn becomes large.
* The term 10n10n10n is smaller and becomes negligible compared to nlog⁡nn \log nnlogn for large nnn.

Thus, f(n)f(n)f(n) behaves like 6nlog⁡n6n \log n6nlogn, which is **Θ(n \log n)**.

**Answer:**  
Yes, f(n)f(n)f(n) is **Θ(n \log n)**.

**Problem 3:**

**Given:**  
f(n)=n3+n2+n+1f(n) = n^3 + n^2 + n + 1f(n)=n3+n2+n+1

**Question:**  
Is f(n)f(n)f(n) **Θ(n³)**?

**Solution:**

We want to determine if f(n)=n3+n2+n+1f(n) = n^3 + n^2 + n + 1f(n)=n3+n2+n+1 is **Θ(n³)**.

* The dominant term in f(n)f(n)f(n) is n3n^3n3, and for large nnn, the n2n^2n2, nnn, and constant terms become negligible.
* Therefore, f(n)f(n)f(n) grows asymptotically like n3n^3n3, which is **Θ(n³)**.

**Answer:**  
Yes, f(n)f(n)f(n) is **Θ(n³)**.

**Summary of Answers:**

**Big Omega (Ω):**

1. **Ω(n²)**: Yes
2. **Ω(n³)**: Yes
3. **Ω(n)**: Yes

**Big Theta (Θ):**

1. **Θ(n²)**: Yes
2. **Θ(n log n)**: Yes
3. **Θ(n³)**: Yes